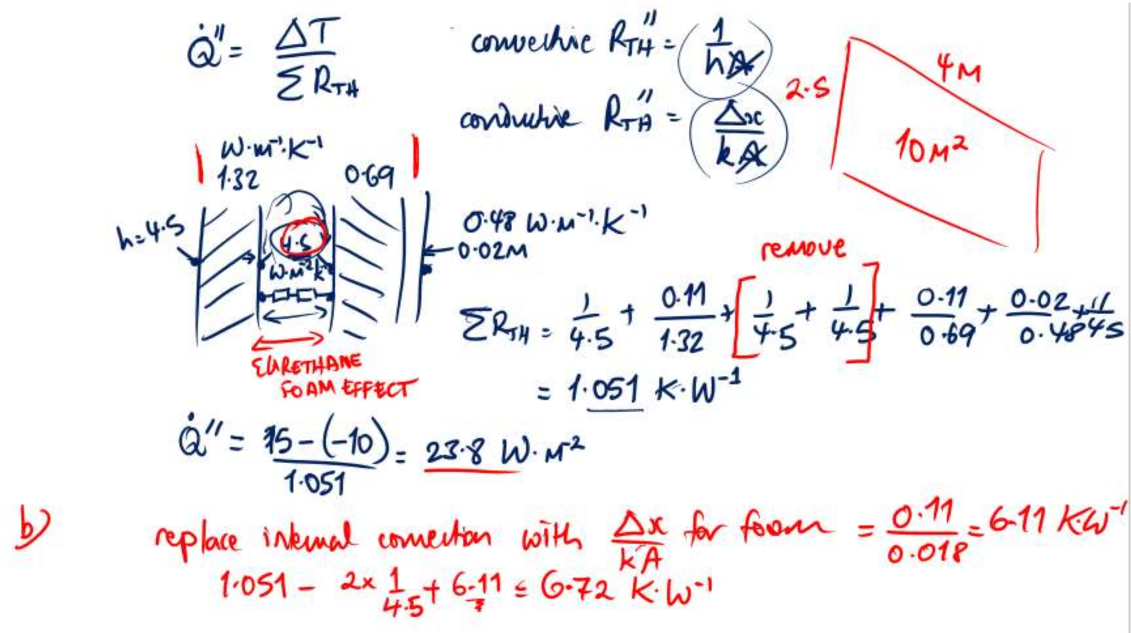
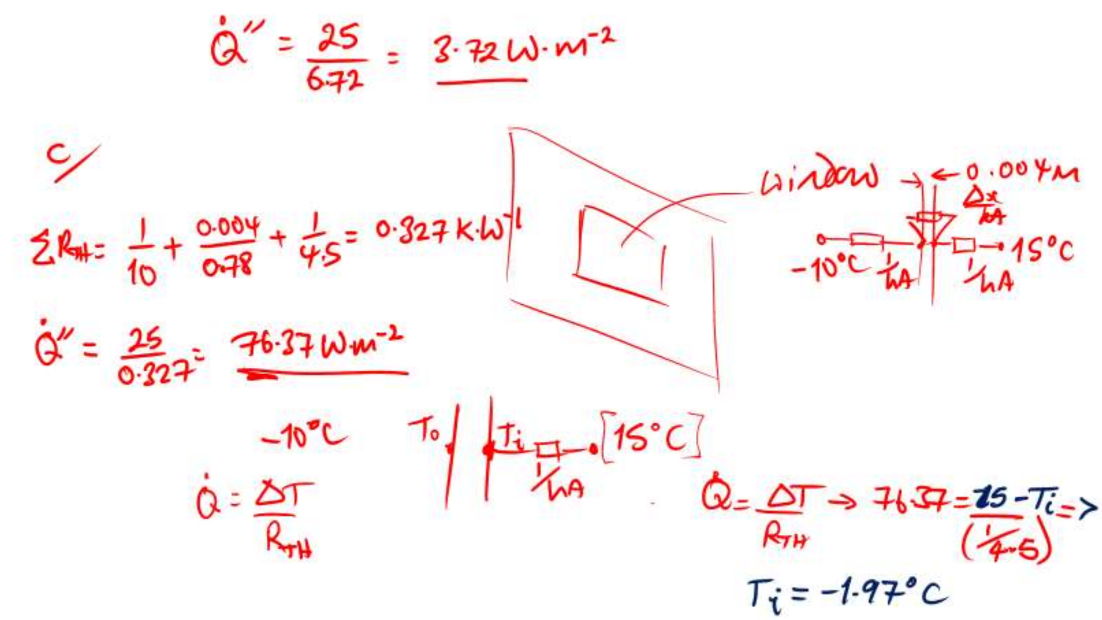
1. This question is a revision question and so is covered in the slides of the video sufficiently





2. Formula for R_{th}

$$R_{th} = \frac{ln\frac{r_0}{r_i}}{2\pi kL} + \frac{1}{hA}$$

the maxima and minima are found by differentiation and finding the zero gradients

$$R_{th} = \frac{\ln\frac{r_0}{r_i}}{2\pi kL} + \frac{1}{hA}$$

therefore:

$$\frac{dR_{th}}{dr_0} = \frac{d}{dr_o}\left(\frac{\ln r_0 - \ln r_i}{2\pi kL}\right) + \frac{d}{dr_o}\left(\frac{1}{2\pi r_o Lh}\right) = \frac{\frac{1}{r_o}}{2\pi kL} - \frac{1}{2\pi {r_o}^2 Lh}$$

For maxima/minima:

$$\frac{\frac{1}{r_o}}{2\pi kL} = \frac{1}{2\pi {r_o}^2 Lh} \rightarrow r_o = \frac{k}{h}$$

this shows that up to a minimum radius, the insulation makes the heat loss worse.

3. Need to calculate Re first which needs density and viscosity from the tables for air – assume that we can use the dynamic viscosity in the tables as not changed much from the 1 atm pressure air because it is still reasonably low pressure:

calculate density using gas law:

$$\rho = \frac{p}{RT} = \frac{200000}{287 \times (260 + 273)} = 1.307$$

units m^3/kg

dynamic viscosity: 2.849×10^{-5} kg/ms

Prandtl by the formula for Pr, with c_p= 1039.8 J/kgK:

$$Pr = \frac{c_p \mu}{k} = 0.69$$

Pr: 0.69

Re = $(1.307 \times 12 \times 0.05)/ 2.849 \times 10^{-5}$ = 27534

Nu = $0.023(27534)^{0.8}(0.69)^{0.4}$ = 70

use Nu = hd/k, k = 0.043 W/mK

h = 60.7 W/m^2K

Then use the thermal resistance combination of the insulation conduction with the internal and external convection, with internal area $2\pi rL = 0.34\ m^2$, and area outer $0.94\ m^2$

$$\frac{1}{hA} + \frac{ln\frac{r_0}{r_i}}{2\pi kL} + \frac{1}{hA} \rightarrow \frac{1}{60.7 \times 0.34} + \frac{ln\frac{0.075}{0.025}}{2\pi \times 0.038 \times 2} + \frac{1}{6.5 \times 0.94} = 0.048 + 2.3 + 0.164 = 2.511$$

units K/W

Therefore

$$\dot{Q} = \frac{260 - 15}{2.511} = 97.6$$

units W

U based on r_i? use the formula relating U and R_{th}

$$UA = \frac{1}{R_{th}} \rightarrow U = \frac{1}{2.511 \times 0.34} = 1.17$$

units W/m²K

4. look for a solution using the Nu correlation and relationship between h and Nu, start with the Gr:

$$Gr = \frac{g\beta L^3 \Delta T}{\nu^2}$$

Find T_{film} (average of surface and far temperature) = (70+15)/2 = 42.5°C or 315 K (approximate to 325 K in tables to save a bit of time in consolidation session, but could interpolate for data)

From tables at 325 K:

$\nu = 1.568 \times 10^{-5}$ m²/s, Pr = 0.71, therefore GrPr =

$$GrPr = \frac{9.81 \times 1/315 \times 0.05^3 (70 - 15)}{1.568 \times 10^2} \times 0.71 = 8547$$

$$\overline{Nu_d} = \left\{ 0.6 + \frac{0.387(8547)^{1/6}}{\left[1 + (0.71)^{9/16}\right]^{8/27}} \right\}^2 = 4.2$$

then use Nu = hd/k, with k = 2.816×10^{-2} W/mK to give:

$$h = \frac{4.2 \times 0.02816}{0.05} = 2.36$$

units are W/m²K

heat transfer is:

$$\dot{Q}' = hA\Delta T = 2.36 \times 2\pi \times 0.006 \times (70 - 15) = 4.9$$

units W/m